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CONVERGING SHOCK WAVES IN MEDIA WITH DECREASING DENSITY

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A number of papers have studied the cumulation of shock waves (see, e.g., [1-8]. Here our specific interest is in investigating shock waves propagating into a decreasing density. For concentric convergent shock waves the problem was solved in [7] by the method of Whitham [6], with which one can evaluate the gas parameters on the wave front in the presence of a piston coming from infinity and generating a continuous inflow of energy in the focus region.

In this paper we compare the solution of [7] with the results of an approximate study of instantaneous energy release (by a strong explosion) at the edge of a closed region with decreasing density of the medium toward the center. We also derived similar solutions for propagation of a spherical shock wave convergent toward the decreasing density in two limiting cases: the adiabatic and the isothermal approximations. The latter regime of the process is linked with the stage of motion when radiative energy transfer appreciably affects the distribution of the flow parameters of the medium. In contrast with [7] the similarity study gave us both the law of motion of the wave front, and the distribution of flow parameters behind the front.

1. We consider propagation of a shock wave toward a decreasing geometric section A and decreasing density of the medium ρ_0 for two limiting laws of energy release at its boundary: 1) exit of a steady strong shock wave generated by a piston moving in from infinity; 2) a strong explosion on a perfectly rigid wall bounding a region with variable A and ρ_0. Physically this means that in the first case the time for the shock wave to focus t_* is much less than the time t^0 for the piston to reach the boundary of the region ($t_* \ll t^0$), and in the second case we have $t_* \gg \tau$ (τ is the duration of the energy release).

In Case 1, applying the rule of characteristics from Whitham [6], we can obtain an equation for the speed of the front of a strong shock wave in a region with decreasing A and ρ_0:

$$d\ln\left(D_1\rho_0^{\beta}A^{\eta}\right)/dx = 0. \tag{1.1}$$

Here x is the coordinate of the front, reckoned from the boundary of the region examined; $\eta = 1/[1 + 2/k + \sqrt{2k/(k-1)}]$; $\beta = 1/[2 + \sqrt{2k/(k-1)}]$; and k is the index of a polytropic medium. Then from the wave front speed D_1, the pressure at the front $p_1 \sim \rho_0 D_1^2$, and the shock wave power $W_1 \sim p_1 D_1$, from Eq. (1.1) we obtain the expressions

$$D_1 \sim \rho_0^{-\beta}(x)A^{-\eta}(x),\ p_1 \sim \rho_0^{1-2\beta}(x)\,A^{-2\eta}(x),\ W_1 \sim \rho_0^{1-3\beta}(x)\,A^{-3\eta}(x). \tag{1.2}$$

As k varies in the range 11/9-3 the corresponding values of the exponents are β = 0.188-0.268, η = 0.148-0.284. This means that the shock speed D_1 increases continuously as the shock propagates, for any laws of decrese of ρ_0 and A. The pressure p_1 and the power W_1 can either decrease, remain steady, or increase, depending on the combination of describing laws for ρ_0 and A, since in the range of k indicated, $1 - 2\beta > 0$ and $1 - 3\beta > 0$.

In case 2 typical parameters of the problem are: surface density of explosive energy is E_0, test region radius is R_0, initial medium density is $\rho_0(R)$, where $R = x_0 - x$ is the radius of the shock wave front, and x_0 is the coordinate of the focus point. In this formu-

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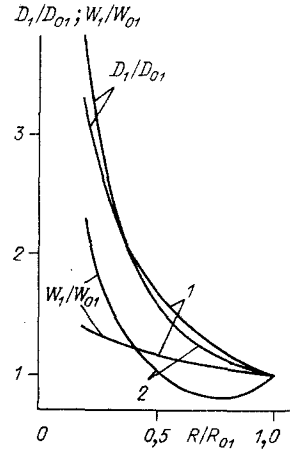

Fig. 1

lation the problem is not self-similar because there is an initial parameter with a dimensioned length. The pressure at the shock wave front is proportional to the mean energy per unit volume [2]. Then, using the known conservation laws at the discontinuity, we can obtain expressions for the speed and strength of a strong shock wave:

$$D_1 \sim [E_0 S_0 (R_0)/\rho_0 (R) V(R)]^{1/2}, \quad W_1 \sim \rho_0 D_1^3 \sim \rho_0^{-1/2}(R) \, [E_0 S_0 (R)/V(R)]^{3/2}. \tag{1.3}$$

Here $S_0(R_0)$ is the area of the energy release surface; and $V(R)$ is the volume of medium set in motion by the shock wave. Analysis of Eq. (1.3) shows that for decreasing $\rho_0(R)$ at some fixed values of the radii R_{01}, R_{02} the corresponding values of D_1 and W_1 reach a minimum, and increase continuously with further propagation of the shock wave. In the case $\rho_0(R) \cong$ const, D_1 and W_1 decrease continuously.

Comparison of the two limiting solutions of Eqs. (1.2) and (1.3), e.g., for spherical symmetry and a power law density decrease $\rho_0 \sim R^{\delta}$, shows that as the focus point is approached the increase of D_1 and W_1 occurs more steeply for a strong shock. This result is illustrated for $\delta = 2$ in Fig. 1, where the curve 1 shows the exit of a stationary shock wave, and curve 2 shows a strong discontinuity. The exponents in Eq. (1.2) were assumed in the form $\beta \cong 0.2$, $\eta \cong 0.2$ as average values in the above range. As the origin we took the values D_{01}, W_{01} at the point $R = R_{01}$, the value at which the shock wave of a strong discontinuity begins to be accelerated.

2. We now consider the problem of a convergent spherically symmetrical shock wave in a gas with density ρ_0 = const r^{δ} decreasing with a power law (r is the ambient radius, $\delta > 0$). In the adiabatic approximation the standard system of gasdynamic equations is written in [5]

$$\begin{aligned} &\partial \ln \rho/\partial t + u\partial \ln \rho/\partial r + \partial u/\partial r + 2u/r = 0, \\ &\partial u/\partial t + u\partial u/\partial r + \rho^{-1}\partial p/\partial r = 0, \\ &\partial \ln (p\rho^{-k})/\partial t + u\partial \ln (p\rho^{-k})/\partial r = 0, \end{aligned} \tag{2.1}$$

where t is time; u, ρ, p are the bulk velocity, density, and pressure of the medium; and k is the adiabatic exponent (polytropic). We introduce the similarity variable $\xi = r/R$ (R is the radius of the shock wave front, assumed as the scale length). We seek a solution of the system (2.1) in the form

$$p = \rho_0 \dot{R}^2 \pi(\xi), \; \rho = \rho_0 g(\xi), \; u = \dot{R} v(\xi). \tag{2.2}$$

Here $\dot{R} = dR/dt$ is the shock speed; and $\pi(\xi)$, $g(\xi)$, $r(\xi)$ are the dimensionless functions. Substituting Eq. (2.2) into Eq. (2.1) and separating the variables t and ξ, we obtain for R and the flow parameters behind the shock the respective equations

$$R = A(-t)^{\alpha}, \; t < 0 \tag{2.3}$$

(A and α are constants);

$$\delta + v' + (v - \xi)(\ln g)' + 2v/\xi = 0, \quad v(\alpha - 1)/\alpha + (v - \xi)v' +$$

TABLE 1

k	δ: 0	1	2	3	4	5
	α					
5/4	0,7443045	0,657853	0,5912589	0,5376453	0,4932809	0,4558393
7/5	0,7171205	0,6265963	0,5587242	0,505025	0,4611551	0,424506
5/3	0,6883547	0,5946639	0,5263143	0,4742293	0,4303797	0,3949006
3	0,6364135	0,5384225	0,4710393	0,420417	0,3804117	0,3477523

TABLE 2

k	δ: 1	2	3	4	5
	α				
5/4	0,5019466	0,4127426	0,3496855	0,302958	0,2670305
7/5	0,4525352	0,3624235	0,3014	0,2575455	0,2246003
5/3	0,404909	0,3155993	0,2577856	0,2174872	0,1878716
3	0,3333333	0,25	0,2	0,1666667	0,1428572

$$+\pi'/g = 0,\ 2(\alpha - 1)/\alpha + \delta(1 - k) + (v - \xi)(\ln(\pi g^{-k}))' = 0 \tag{2.4}$$

(the prime denotes differentiation with respect to ξ). We determine the boundary conditions for the system of equations obtained with $\xi = 1$ from Eq. (2.2), starting from the relations at the front of a strong shock wave:

$$\pi(1) = 2/(k+1),\ g(1) = (k+1)/(k-1),\ v(1) = 2/(k+1).$$

In Eq. (2.4) the unknown parameter is the similarity exponent α, and the values of δ and k are given from the conditions of the problem. The quantity α, which determines the law of motion of the wave front, Eq. (2.3), is found from the condition that the solution of the system of equations (2.4) be unique.

Introducing the variables $p(\xi) = \alpha^2\pi(\xi)/\xi^2$, $V(\xi) = \alpha v(\xi)/\xi$, $\sigma(\xi) = g(\xi)$, $Z = kp/\sigma$, and transforming the system of equations (2.4) and the corresponding boundary conditions, and choosing α numerically to satisfy the condition [1] that the integral curve $Z(V)$ should pass through the special point Z^*, V^*, where the determinant of the system of equations obtained goes to zero simultaneously, we find the solution of this problem to within an undefined multiplier A. Here the distribution of gas density in the flow is found accurately from the fact that the initial law $\rho_0(r)$ is given from the condition, while the density ρ_1 at the front of a strong shock is given from the relation $\rho_1 = \rho_0(k + 1)/(k - 1)$.

In the isothermal approximation ($\partial T/\partial r = 0$, T is the temperature) the system of gasdynamic equations for a polytropic gas has the form

$$\begin{aligned}&\partial \ln\rho/\partial t + u\partial \ln\rho/\partial r + \partial u/\partial r + 2u/r = 0,\\ &\partial u/\partial t + u\partial u/\partial r + (p/\rho^2)\partial p/\partial r = 0.\end{aligned} \tag{2.5}$$

In contrast to [8] in this paper we consider a convergent spherically symmetric shock wave. The method of solution is analogous to that described above. Introducing the variables $\xi = r/R$, v, g, π, we find $u = \dot{R}v(\xi)2/(k + 1)$, $\rho = \rho_0 g(\xi)(k + 1)/(k - 1)$, $p = \rho_0\dot{R}^2\pi(\xi)2/(k + 1)$ with the boundary conditions $v(1) = g(1) = \pi(1) = 1$. For the similarity distributions of parameters behind the shock from Eq. (2.5) we find the system of equations

$$\begin{aligned}&v(\alpha-1)/\alpha + [2v/(k+1) - \xi]dv/d\xi + [(k-1)/(k+1)]d\ln g/d\xi = 0,\\ &\delta + [2v/(k+1) - \xi]d\ln g/d\xi + [2/(k+1)]dv/d\xi + [4/(k+1)]v/\xi = 0.\end{aligned} \tag{2.6}$$

Making the substitution $\varkappa = 1/\xi$ for the functions v and g we have

$$\begin{aligned}&\frac{dv}{d\varkappa} = \frac{\delta\varkappa(k-1)/(k+1) + 4v\varkappa^2(k-1)/(k+1)^2 - [2v\varkappa/(k+1) - 1]\,v(\alpha-1)/\alpha}{\varkappa\{\varkappa^2 2(k-1)/(k+1)^2 - [2\varkappa v/(k+1) - 1]^2\}},\\ &\frac{d\ln g}{d\varkappa} = \frac{4v\varkappa/(k+1)[2v\varkappa/(k+1) - 1] + 2v\varkappa(1-\alpha)/\alpha(k+1) + \delta[2v\varkappa/(k+1) - 1]}{\varkappa\{[2v\varkappa/(k+1) - 1]^2 - 2(k-1)\varkappa^2/(k+1)^2\}}.\end{aligned} \tag{2.7}$$

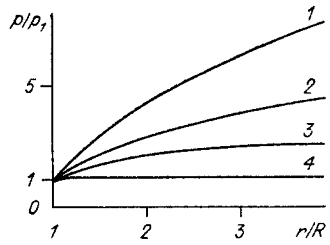

Fig. 2

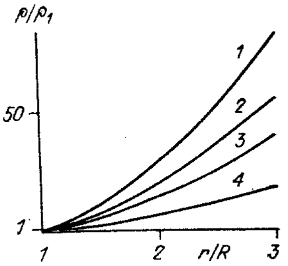

Fig. 3

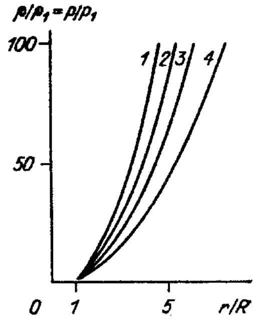

Fig. 4

The parameter α in the law of motion of the shock front (2.3) and in the system (2.6) is determined from the condition that the integral curve should pass through the singular point of Eq. (2.7):

$$\varkappa^* = \frac{2(1-\alpha)/\alpha}{\sqrt{2(k-1)/(k+1)^2}\left\{\sqrt{[2+\delta+(1-\alpha)/\alpha]^2 - 8(1-\alpha)/\alpha} + 2+\delta+(1-\alpha)/\alpha\right\}},$$

$$v^* = \frac{2+\delta+(\alpha-1)/\alpha + \sqrt{[2+\delta+(\alpha-1)/\alpha]^2 + 4(1-\alpha)/\alpha}}{[2(1-\alpha)/\alpha]\sqrt{(k-1)/2}}.$$

Just as in the adiabatic approximation the pressure and the velocity are found to within an undetermined constant, this being linked to the indeterminacy of the factor A in Eq. (2.3).

The computed values of the similarity exponent α in the adiabatic and isothermal approximations are shown in Tables 1 and 2, respectively. As in the planar symmetry case [8] in the isothermal approximation α is less than in the adiabatic approximation, i.e., the speed $\dot{R} \sim R^{(\alpha-1)/\alpha}$ of the front "isothermal" shock increases more steeply as one approaches the center of symmetry than does the front "adiabatic" shock. This behavior of the shock is linked to the large pressure gradient behind the front $\partial p/\partial r$ for $\partial T/\partial r = 0$, which can be seen by comparing the distributions of p and ρ in the flow presented in Figs. 2-4 for $\delta = 2$. The origin for the computation was the values of the parameters on the front: $r/R = 1$, $\rho/\rho_1 = p/p_1 = 1$. Curves 1-4 are for k = 5/4, 7/5, 5/3, 3. Figures 2 and 3 correspond to the adiabatic approximation, and 4 to the isothermal.

3. The results shown in Tables 1 and 2 may prove useful in the choice of experimental conditions to investigate convergent shock waves in media with decreasing density, and also the action of these shock waves on targets located at the center of the focusing region. Thus, by comparing the solutions (1.2) and (1.3) (see Fig. 1) we can conclude that a shock wave formed at the strong discontinuity on the boundary of a region with decreasing density is more restricted in radiative time and also in mechanical action on the target than the shock wave generated by a piston. Of specific interest is the pressure distribution of Fig. 2 in a flow with k = 3, $\delta = 2$. The pressure behind the shock is almost independent of the coordinate, i.e., is constant. Computations show that for $\delta > 2$ the pressure gradient behind the shock is positive ($\partial p/\partial r > 0$) for k = 3. This type of p distribution generates favorable conditions for compression of targets, e.g., in the study of phase transformations and creating new materials.

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ALLOWING FOR INTERMOLECULAR ENERGY EXCHANGE IN THE DESCRIPTION OF RELAXATION PROCESSES IN TERMS OF ADIABATIC VARIABLES

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The solution of the complete system of gasdynamic equations, supplemented by kinetic equations describing vibrational-rotational relaxation or the kinetics of phase transformations [1-4], is an exceedingly complex problem. The difficulty of the problem makes it important to find simpler methods of describing transformation kinetics by means of approximate solutions of relaxation equations [1, 5]. One such method was developed in [6-9] for rate coefficients of arbitrary form. It is based on the introduction of adiabatic variables which diagonalize (in the case of distributions which are smooth with respect to quantum number) the initial system of kinetic equations to within a small parameter.

In the present study – a continuation of [6-9] – we examine two methods of describing the contribution of intermolecular energy exchange to the relaxation of the populations of individual levels in terms of adiabatic variables. The methods make it possible to obtain approximate analytic solutions of the kinetic equations for different relaxation regimes.

1. System of Relaxation Equations and Adiabatic Variables. Let us examine the process of relaxation in a mixture of molecules of species s. We will assume that the internal state of a molecule is characterized by a single quantum number ν (such as in the case of delayed vibrational relaxation in a mixture of diatomic molecules [10]). The equations for the populations of individual energy levels $n_s(\nu) \equiv n_s(\nu; \mathbf{r}, t)$ have the form [10]

$$\frac{\partial n_s(\nu)}{\partial t} + \nabla\cdot[\mathbf{u}n_s(\nu)] + \nabla\cdot[\mathbf{u}_s(\nu)n_s(\nu)] = I_\nu(s\,|\,\mathbf{n}),$$
$$I_\nu(s\,|\,\mathbf{n}) = \sum_{i=1,2} I_\nu^{(i)}(s\,|\,\mathbf{n}) = \sum_{i=1,2}\sum_{s_1} I_\nu^{(i)}(s, s_1\,|\,\mathbf{n}), \tag{1.1}$$

where $\mathbf{u}$ is the hydrodynamic velocity; $\mathbf{u}_s(\nu)$ is the rate of diffusion of molecules of species s in the state ν; $I_\nu^{(1)}$ is the linear part of the collision integral, describing the transfer of energy between the internal and translational degrees of freedom of the gas; and $I_\nu^{(2)}$ is the quadratic part of the collision integral, responsible for intermolecular energy transfer.

For the concentrations

$$x_s(\nu) \equiv n_s(\nu)/n,\ \ n \equiv \sum_{s,\nu} n_s(\nu) \tag{1.2}$$

Eqs. (1.1) take the form

$$\dot{x}_s(\nu) + n^{-1}\nabla\cdot[\mathbf{u}_s(\nu)n_s(\nu)] = I_\nu(s|\mathbf{x}) \tag{1.3}$$

at

$$I_\nu^{(1)}(s, s_1\,|\,\mathbf{x}) = \sum_\mu [P_{\mu\nu}(s, s_1)\,x_s(\mu) - P_{\nu\mu}(s, s_1)\,x_s(\nu)],$$
$$I_\nu^{(2)}(s, s_1\,|\,\mathbf{x}) = \sum_{\varkappa,\lambda,\mu} \left[P_{\mu\nu}^{\varkappa\lambda}(s, s_1)\,x_s(\mu)\,x_{s_1}(\varkappa) - P_{\nu\mu}^{\lambda\varkappa}(s, s_1)\,x_s(\nu)\,x_{s_1}(\lambda)\right] \tag{1.4}$$

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